PHYSICS 536
Experiment 13: Active Filters

Active filters provide a sudden change in signal amplitude for a small change in frequency. Several filters can be used in series to increase the attenuation outside the break frequency. High-pass, low-pass, band-pass, Butterworth, and Chebyshev filters are investigated in this experiment.

Two methods of filter design are considered. a) Convenient time-constants are selected. Then the closed-loop gain (G_o) is adjusted to obtain the desired frequency response. b) The closed-loop gain is selected, and then the time constants are adjusted to obtain the desired frequency response.

A. **The Low-Pass Filter** circuit is shown in the following sketch:

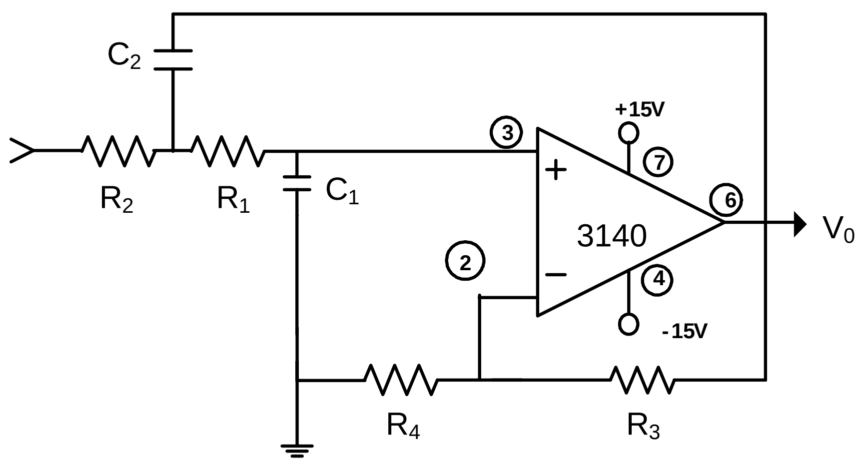

Figure 1

The closed-loop gain is

$$G_o = (R_3 + R_4)/R_4 \tag{13.1}$$

1) **Gain adjustment method.** Set $R_1 = R_2$ and $C_1 = C_2$. A convenient capacitor is selected and the resistor size is calculated to obtain the desired break frequency f_c.

$$R_1 = (2\pi f_n f_c C_1)^{-1} \tag{13.2}$$

Next R_4 is selected for convenience, and R_3 is calculated to adjust G_o to the value needed to obtain the desired frequency response (i.e., Bessel, Butterworth, Chebyshev, etc.).

$$R_3 = R_4(G_o - 1) \tag{13.3}$$

G_o and f_n are obtained from Table 5.2 in the text. G_o is labeled as "K" in the table. f_n is one for a Butterworth filter.

Multistage Filters. Better filter performance can be obtained by including more R-C attenuators. (Each R-C attenuator contributes one "pole" to the filter.) Only two R-C attenuators can be used with one op-amp, hence better filters have several op-amps in series. For example, an 8-pole filter would use 4 op-amps. The individual op-amps in the filter do not have the same frequency response as a 2-pole filter of the same type, as shown by the parameters in Text Table 5.2. Each op-amp in a multistage filter has a different frequency response, but their combined effect is closer to an ideal filter, i.e. no attenuation below the break and no gain above the break. The gain expression is simple for a multistage Butterworth Filter.

$$G = G_o\left[(f / f_c)^{2n} + 1)\right]^{-1/2} \tag{13.4}$$

n is the number of poles in the total filter.

The gain is approximately constant ($G = G_o$) below the break frequency (f_c), because the term (f / f_c) is very small when n is large and $f < f_c$. The same term makes the gain decrease rapidly when $f > f_c$.

2) **Time-constant adjustment method.** The gain can be set to any convenient value. In this example, G_o =1 is obtained by eliminating R_4 and using a direct connection for R_3. Then the filter acts as a "follower" below the break frequency. For a 2 pole Butterworth filter we can use:

3)

$C_2 = 2C_1$ and $R_1 = R_2$

Then

$$R_1 = (2\pi f_c C_1 \sqrt{2})^{-1} \tag{13.5}$$

B. **The High-Pass Filter** circuit is shown in the following sketch:

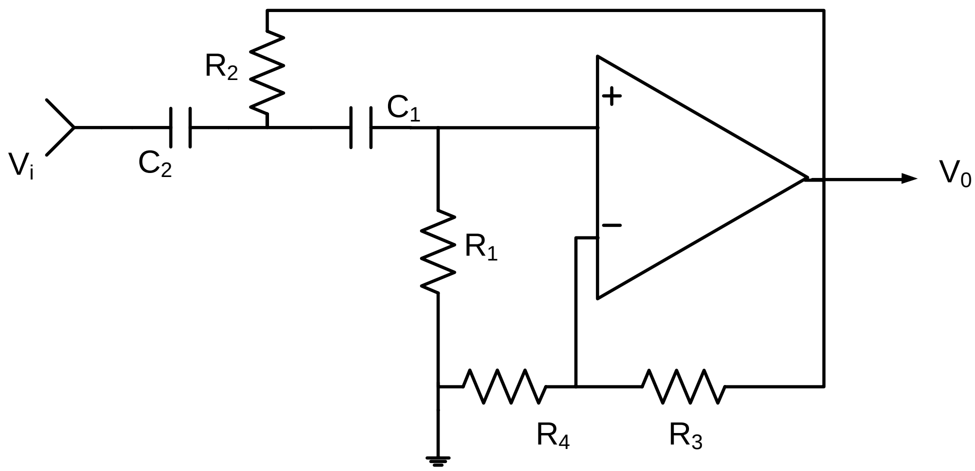

Figure 2

1) Gain adjustment method. Set $R_1 = R_2$ and $C_1 = C_2$. A convenient capacitor size is selected and the resistor size is calculated to obtain the desired break-frequency.

$$R_1 = \left[2\pi f_n^{-1} f_c C_1\right]^{-1} \tag{13.6}$$

The gain is the same as the low-pass case.

$$R_3 = R_4(G_o - 1) \tag{13.7}$$

G_o (=K) and f_n are taken from Table 5.2 of the test. The (f_c / f) term in the Butterworth gain expression is inverted relatively to the low pass filter.

$$G = G_o\left[(f_c / f)^{2n} + 1\right]^{-1/2} \tag{13.8}$$

where n is the total number of poles in the filter.

In this case, the gain drops rapidly when f is smaller than the break frequency.

C. **The Band-Pass Filter** is shown in the following sketch:

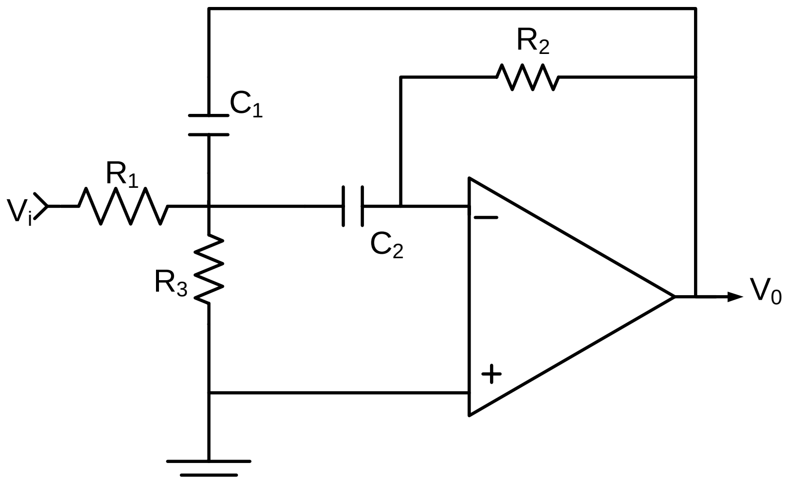

Figure 3

The gain of this circuit is maximum (G_r) at the resonance frequency f_r.

$$G_r = \frac{R_2}{R_1}\frac{C_2}{C_1 + C_2}\frac{1}{1 + G_o / A} \tag{13.9}$$

$$f_r^{\ 2} = \frac{1 + R_1 / R_3}{(2\pi)^2 \tau_1 \tau_2}\frac{1 + G_o / A}{1 - C_2 /(C_1 A \angle 90^\circ)} \tag{13.10}$$

$$\tau_j = R_j C_j \tag{13.11}$$

A is the open-loop gain of the op-amp at the resonance frequency,

$$A = f_T / f_r \tag{13.12}$$

f_T is the gain-frequency product of the op-amp.

$$G_o = R_2 / X_{c2} = 2\pi f_r C_2 R_2 \tag{13.13}$$

When the open-loop gain is large, the two terms containing A^{-1} can be neglected.

$$f_r^{\,2} = \frac{1 + R_1 / R_3}{(2\pi)^2 \tau_1 \tau_2} \tag{13.14}$$

Therefore, the two time constants set the minimum resonance frequency, but f_r can be increased by making R_3 smaller than R_1.

The band-pass (B) is defined as the frequency interval between the high and low break-frequencies (i.e., the frequencies at which gain is decreased by 30%).

$$\text{B(Hz)} = \left[2\pi R_2 C_1 C_2 /(C_1 + C_2)\right]^{-1} \tag{13.15}$$

A CA3140 op-amp will be used for all circuits. Positive and negative 15V power supplies with by-pass capacitors should be connected to each circuit.

A. Low-Pass Filters:

1) Homework Gain-adjustment method for a two-pole Butterworth filter. Use the values of R_4, C_1, and C_2 given at the back of these instructions to calculate the values of R_1, R_2, and R_3 needed to obtain the specified break frequency (f_c). Calculate the gain of the filter at 0.5, 1, 2, 4, and 10 times f_c.

2) Homework Measure the break frequency and the gain at 0.5, 2, 4, and 10 times the **measured** break frequency. Use a 10V(p-p) input signal.

3) Homework Time-constant adjustment method for a two-pole, G_o = 1, Butterworth filter. Use the value of C_1 given and calculate the values of R_1, R_2, and C_2 needed to obtain the specified f_c and Butterworth frequency response. Calculate G at 0.5, 1, 2, 4, and 10 f_c.

4) Homework Set up the circuit designed in step 3. Measure the break frequency and the gain at 0.5, 2, 4, and 10 times the **measured** break frequency. Use a 10V(p-p) input signal.

5) Homework Gain-adjustment method for a four-pole Chebyshev (0.5db) filter. Use the components given at the back of these instructions. Calculate R_1, R_2, and R_3 for both stages and the total gain. You are not required to build and test this circuit.

B. High-Pass Filters.

6) Homework Gain-adjustment method for a two-pole Butterworth filter. Use the values of R_4, C_1, and C_2 given to calculate the values of R_1, R_2, and R_3 needed to obtain the specified break frequency. Calculate the gain of the filter at 2, 1, 0.5, 0.25, and 0.1 times f_c.

7) Homework Set up the circuit designed in step 6. Measure the break frequency and the gain at 2, 0.5, 0.25, and 0.1 times the **measured** break frequency. Use a 5V(p-p) input signal.

C. Band-Pass Filters.

8) Homework Assume that the open-loop gain is sufficiently large that terms with A^{-1} can be neglected. Calculate the approximate resonance frequency f_r. Use this approximation to calculate f_r including the A^{-1} terms. Calculate the gain at resonance (G_r) and the band-width (B).

9) Homework Set up the circuit described in step 10. Measure f_r, G_r, and B. B is the interval between the two frequencies at which G = 0.7G_r. The input signal should be small so that v_o will not be too large at resonance. v_i = 20mV(p-p) is appropriate. You should adjust the signal generator to keep the amplitude of v_i constant when the frequency is changed (GI-4.3). The amplitude of v_i can change because the input resistance of the circuit is frequency dependent. A 10% disagreement between calculation and measurement can easily occur because R and C values are not precise.

10) Homework The resonance will be shifted to higher frequency by including R_3. Calculate the change in f_r.

11) Homework Add R_3 and measure f_r and G_r. The decrease in G_r is produced by inaccurate components and a lower quality factor (A/G_o).

Physics 536: Experiment 9(A)

X = Not included. D = Direct connection C = Calculate the value
R = Read from text Table 5.2 f_t = 3.7MHz for the CA3140

Step	R_1	R_2	R_3	R_4	C_1 (nf)	C_2 (nf)	f_c (kHz)	f_n	G_o
1-2	C	C	C	5.1K	1	1	12	1	R
3-4	C	C	D	X	1	C	12	1	1
5	C	C	C	5.1K	1	1	12	R	R
6-7	C	C	C	5.1K	1	1	12	1	R
8-9	1K	100K	X	X	5	10	--	--	--
10-11	1K	100K	200 Ω	X	5	10	--	--	--

COMPONENTS

Semiconductor: CA3140
Resistors: 3K, 5.1K, 2-13K
Capacitors: 2-0.1 μ f, 2-1nf

Please pick up the following components later if someone is waiting.
Resistors: 200 Ω , 1K, 2-9.1K, 100K
Capacitors: 2-2nf, 5nf, 10nf

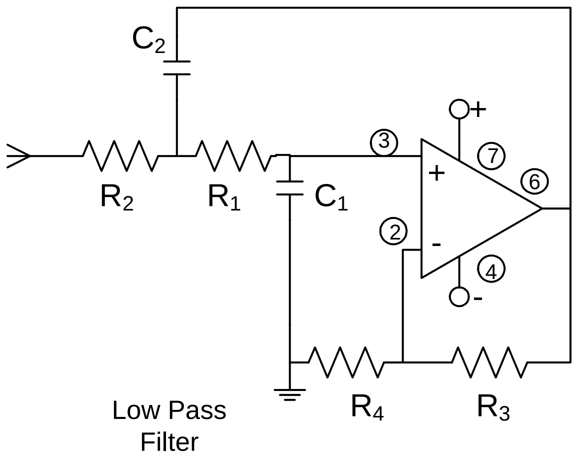

Figure 4

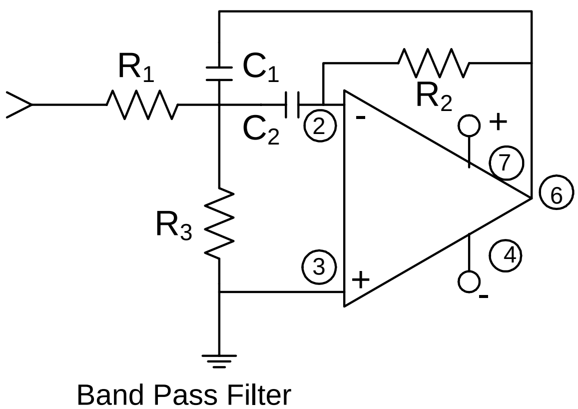

Figure 5

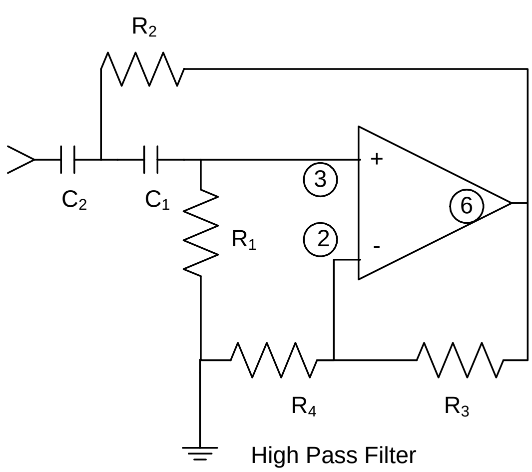

Figure 6